The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Robert William Chapman, M.A., B.C.E., Lecturer on Engineering and Physical Science in the University, Adelaide, South Australia (proposed by Sir C. Todd) ; and
Charles J. Isaacs, Head of the Upper Nautical School, Greenwich, S.E. (proposed by Thomas Lewis).

Fifty-three presents were announced as having been received since the last meeting, including, amongst others :—

Greenwich Second Ten-year Catalogue of 6892 stars for the epoch 1890·0, presented by the Observatory ; Series of twenty-six Charts of Borneo, Sumatra, &c., for the use of observers of the total solar eclipse of May 1901, presented by Professor Bakhuyzen.

On mechanically compensating the rotation of the field of a Siderostat. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In the *Astrophysical Journal* for March 1900, M. Cornu has given an elegant investigation of the rotation of the siderostat field. To the geometry of his paper little or nothing can be added ; but he has left untouched the question of any mechanical compensation of the variable rotation described, of which I propose to treat in the present note. Some simple link-work arrangements are suggested which it is hoped may be found to solve the problem of compensation practically.

2. Let P (in fig. 1) denote the pole of the heavens ; T the direction in which the telescope is directed, looking from mirror through the OG to the eye end ; and draw the meridian PT, which is not necessarily the meridian of the zenith but that of the telescope.

Consider what happens as a star S passes from upper culmination (on the *telescope* meridian) at S_1 to lower culmination at S_2. If the siderostat reflects its image in the constant direction T, then at upper culmination the normal to the mirror is directed towards N_1 where N_1 bisects the arc TS_1. The pole P will thus be reflected to R_1 *below* T ; and since $PN_1 = N_1R_1$, we have $TR_1 = PS_1$. Similarly at lower culmination the pole is reflected to R_2. And it is easily seen geometrically, as is shown in

M. Cornu's paper, that the image of P describes a circle about T, with radius TR equal to the radius PS, of the circle described by the star about P. The essential difference between the description of the two equal circles is that S moves round P uniformly, but R does *not* move round T uniformly.

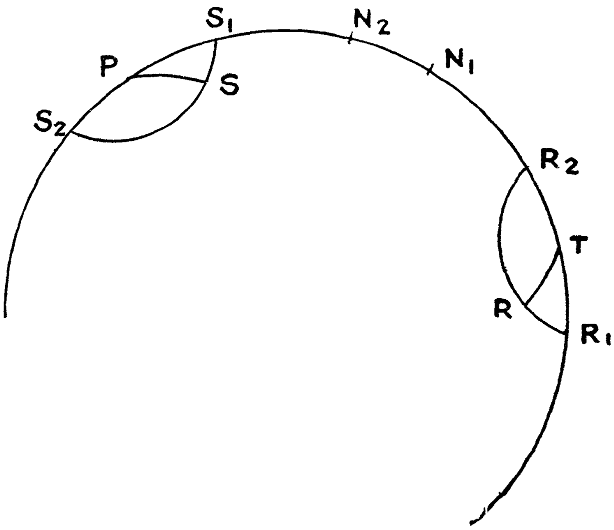

FIG. 1.

3. Its motion is given by the equation

$$\tan \tfrac{1}{2}\mathrm{RTR}_1 = \mathrm{K} \tan \tfrac{1}{2}\mathrm{SPS}_1 \qquad \ldots \qquad \ldots \qquad (1)$$

where K is a constant depending simply on the distances PS and PT. If PS $=\delta$ and PT $=\rho$, then M. Cornu shows* that

$$\mathrm{K} = \frac{\cos \frac{1}{2}(\rho+\delta)}{\cos \frac{1}{2}(\rho-\delta)} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

Thus when $\mathrm{SPS}_1 = 0°$ or $180°$, $\mathrm{RTR}_1 = 0°$ or $180°$, but for intermediate values the angles differ. [For the cœlostat $\rho+\delta=180°$; hence $\mathrm{K}=0$ and $\mathrm{RTR}_1 = 0°$ permanently, except when $\mathrm{SPS}_1 = 180°$. This case is more particularly referred to in § 12.]

4. Now the motion of TR defines the rotational motion required. If we suppose a photographic plate pivoted at T, and moving round T as if TR were rigidly attached to the plate, then all the rest of the picture will remain fixed relatively to the plate, which is the condition required for photography. We proceed to consider various mechanical ways of securing this motion.

5. *First Device.*—If S and C be fixed points, and PN a fixed line at right angles to them on which the point P moves, then

* M. Cornu finds the reciprocal of this expression; but I think there is an algebraical slip in passing from one equation to the other on p. 151 *loc. cit.*

$$\tan \mathrm{PSN} = \frac{\mathrm{PN}}{\mathrm{SN}}$$

$$= \frac{\mathrm{CN}}{\mathrm{SN}} \cdot \frac{\mathrm{PN}}{\mathrm{CN}}$$

or

$$\tan \mathrm{PSN} = \frac{\mathrm{CN}}{\mathrm{SN}} \tan \mathrm{PCN} \quad \ldots \quad \ldots \quad (3)$$

Thus, if

$$\frac{\mathrm{CN}}{\mathrm{SN}} = \mathrm{K} = \frac{\cos \frac{1}{2}(\rho + \delta)}{\cos \frac{1}{2}(\rho - \delta)} \quad \ldots \quad \ldots \quad (4)$$

and if the angle PCN is made the same as $\frac{1}{2}\mathrm{SPS}_1$ in equation (1), and therefore increases uniformly with the time, then PSN will be the same as $\frac{1}{2}\mathrm{RTR}_1$, and we have only to make another line move at twice the rate of SP.

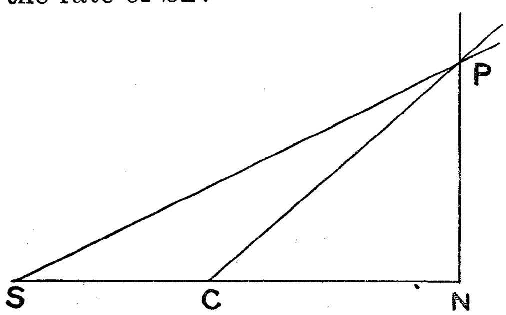

FIG. 2.

Mechanically, SCN and NP must be two bars, PN being grooved. CP and SP must be bars pivoted at S and C, and also grooved; and a pin, P, must connect all three together. CP is to be rotated uniformly by clockwork once in two days, and then SP will move nearly as required. To complete the apparatus we must make another bar or wheel move just twice as fast as SP, which can be done in a variety of ways—*e.g.* by cog-wheel gearing in the ratio of 2 to 1.

6. *Second Device.*—The equation

$$\tan \theta = \mathrm{K} \tan nt \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

is really a particular case of epicyclical motion. Let a point A (fig. 3) be revolving uniformly round a point O at unit distance from it, so that its coordinates may be written

$$x = \cos u \qquad y = \sin u.$$

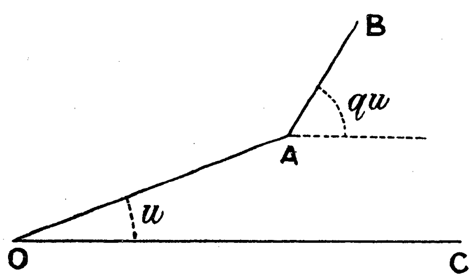

FIG. 3.

Let a second point, B, be revolving uniformly about A, so that its coordinates *referred to* A are

$$x' = p\cos qu \qquad y' = p\sin qu.$$

Then the coordinates of B with respect to O will be

$$x + x' = \cos u + p\cos qu \qquad y + y' = \sin u + p\sin qu\ ;$$

and the direction θ in which B is seen from O is given by the familiar equation

$$\tan\theta = \frac{y+y'}{x+x'} = \frac{\sin u + p\sin qu}{\cos u + p\cos qu} \ldots \qquad \ldots \quad (6)$$

This is the general equation for epicyclical motion.

7. If $q = -1$, we have

$$\tan\theta = \frac{\sin u - p\sin u}{\cos u + p\cos u} = \frac{1-p}{1+p}\tan u \qquad \ldots \quad (7)$$

which resembles equations (1), (3), (5).

In this case the two revolutions are performed in the same period in opposite directions, and the curve is easily seen to be an ellipse.

For the coordinates of B are

$$x + x' = (1+p)\cos u$$

and $$y + y' = (1-p)\sin u,$$

which represent an ellipse, axes $2(1+p)$ and $2(1-p)$.

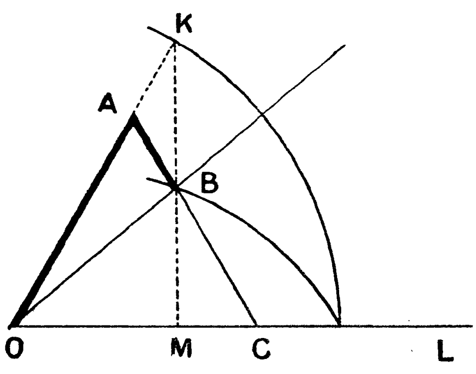

Fig. 4.

Thus let OA, AC be two equal bars, and let OA describe a circle round O uniformly, and let AC be pivoted at A, and the point C constrained to move along the line OL, so that OAC is an isosceles triangle, and AC revolves round A uniformly at the same pace as AO round O, but in the opposite direction. Then any point B on AC, where

$$AB = p \,.\, OA,$$

describes an ellipse ; and further, if a bar, OB, be freely pivoted at O and constrained to pass through B, it will move so that

$$\tan \mathrm{BOL} = \frac{1-p}{1+p} \tan \mathrm{AOL} \quad \ldots \quad \ldots \quad (8)$$

It is easily seen that if OA be produced to K, where

$$\mathrm{AK}=\mathrm{AB},$$

then K lies on the ordinate MB, and also on the auxiliary circle ; and equation (8) is only another form of the well known relation

$$\mathrm{KM} : \mathrm{BM} = 1+p : 1-p,$$

i.e. in the ratio of the semi-axes.

8. These properties indicate a simple form of ellipsograph, which is probably not new,* but which I do not happen to have seen. It is, however, closely related to an ordinary form. If OL and ON be two fixed bars at right angles, and CD be a bar of constant length whose ends, C, D, slide on OL and ON respectively, then any point B on CD is known to describe an ellipse. If A be the middle point of CD, then OA=AC=AD,

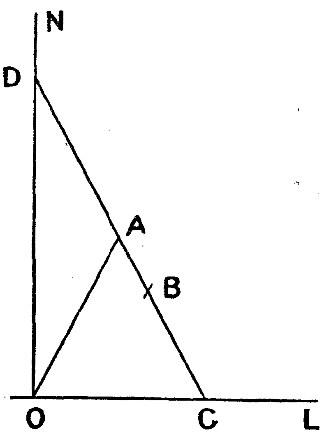

FIG. 5.

and is of constant length ; and we may remove the portion AD if we tie A to O by a bar, which gives the above construction.

9. Having secured the motion of the bar OB so as to satisfy the equation

$$\tan \mathrm{BOL} = \frac{1-p}{1+p} \tan \mathrm{AOC},$$

it remains only to make the bar AO revolve once in two days, as

* Mr. Wesley, after the reading of this paper, drew my attention to the fact that this form of ellipsograph has been described by Mr. Burnham in *Popular Astronomy*, vol. iv. p. 181.

in the first device, and to attach a cog-wheel or other arrangement to OA, so as to double its angular motion. The advantage of this arrangement over the first is that all the bars remain of constant length. In fig. 2 it will be seen that as P travels up NP the bars SP and CP must be longer and longer; and indeed *complete* motion with the first arrangement is impossible. Since we should in practice, however, usually be working near the meridian of the telescope, the first device might be found useful in spite of its disadvantages.

10. *Third Device.*—But there is a third arrangement which is prettier (geometrically) than either. In both the former devices we have had to multiply the movement by 2, by a pair of cog-wheels or some other multiplying arrangement. In fact, we have considered the equation

$$\tan \theta = \mathrm{K} \tan nt \qquad \ldots \qquad \ldots \qquad (9)$$

rather than the equation

$$\tan \tfrac{1}{2}\theta = \mathrm{K} \tan \tfrac{1}{2}nt \qquad \ldots \qquad \ldots \qquad (10)$$

But equation (10) is a familiar one. It represents motion about the focus of an ellipse just as equation (9) represents motion about the centre. In "Dynamics of a Particle" we have the equation

$$\tan \tfrac{1}{2}\theta = \sqrt{\frac{1-e}{1+e}} \,.\, \tan \tfrac{1}{2}u \qquad \ldots \qquad (11)$$

where u is the eccentric anomaly, and θ the true anomaly. Thus in fig. 6 let, as before, two equal bars OA, AC be pivoted at O and A, and let C be free to slide along SOL. Then any point B in AC will describe an ellipse. Let S be the focus of this ellipse, and let a bar pivoted at S be constrained to pass through B. Then it will move in the required manner *without* any multiplying device, if OA revolve uniformly *once* a day. I think this seems the best way of mechanically compensating the rotation.

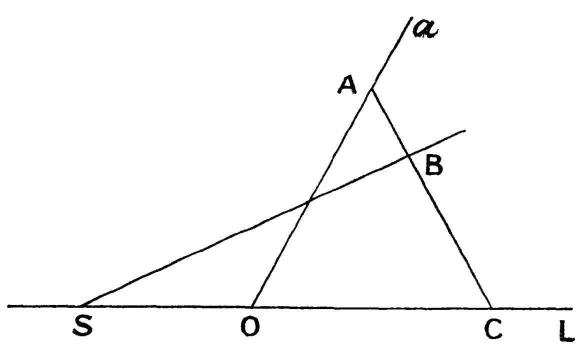

FIG. 6.

11. As regards details, since the unit of length is arbitrary, let us first consider keeping the points O and S fixed. There is

a convenience about this, since S is practically the centre of the plate. The pivot at A must then be altered, and the length of AC adjusted, as well as the point B, to suit the particular star. It seems better to keep OA and AC fixed, and to adjust B and S. To find their positions we have

$$\sqrt{\frac{1-e}{1+e}}=K=\frac{\cos\frac{1}{2}(\rho+\delta)}{\cos\frac{1}{2}(\rho-\delta)} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$\therefore \frac{1-e}{1+e}=\frac{(1+\cos\rho\cos\delta)-\sin\rho\sin\delta}{(1+\cos\rho\cos\delta)+\sin\rho\sin\delta}$$

or

$$e=\frac{\sin\rho\sin\delta}{1+\cos\rho\cos\delta} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Further, if $AB=p \cdot AO$, the axes are $1+p$ and $1-p$; or

$$(1+p)^2(1-e^2)=(1-p)^2$$

or

$$e^2=\frac{4p}{(1+p)^2} \quad \ldots \quad \ldots \quad (14)$$

Hence, $p=2-e^2+2\sqrt{1-e^2}=(1+\sqrt{1-e^2})^2 \quad \ldots \quad \ldots \quad (15)$

and $OS=e(1+p)=2\sqrt{p}=2+2\sqrt{1-e^2} \quad \ldots \quad \ldots \quad \ldots \quad (16)$

12. *The Cœlostat.*—The particular case of the Cœlostat deserves notice. In § 2 it was remarked that the reflected image R of the Pole always describes a circle about T of radius equal to PS—*i.e.* of finite radius, depending only on the star. And yet in the Cœlostat we know that the image of the Pole, like every other point of the sky, remains fixed. How is this apparent paradox to be explained? The above construction shows the way in which it occurs as a limiting case. The limit is when $\cos\frac{1}{2}(\rho+\delta)=0$, and $e=1$. The ellipse thus becomes a parabola, and S is at an infinite distance.

But suppose that, as suggested in the beginning of the last paragraph, we keep the length OS constant, and vary OA and AC. As e approaches unity, p approaches unity—*i.e.* B approaches very near to C; and its motion becomes nearly the same as that of C—*i.e.* it travels nearly along the straight line LO. Thus SB remains nearly coincident with SC, *until* B gets near S. Then SB travels very swiftly round through nearly 360°, and comes to approximate coincidence with SC, but on the other side.

Thus the limiting case of the cœlostat is one in which the plate remains fixed for the whole twenty-four hours less one instant, in which time it suddenly makes a complete revolution. The arc TR still describes its circle, but in *no time.*

It will now be clear how the cœlostat occupies the position of transition between cases where TR describes its circle in opposite directions. The revolution in *no time*—*i.e.* with infinite velocity—may be performed indifferently in either direction

just as asymptotes meet a curve at infinity in either direction. If this explanation is correct, the case is a curiosity which has (so far as I know) hitherto escaped attention.

Observations of Saturn made at Juvisy Observatory in 1900.
By C. Flammarion.

The planet was observed from July to October last, and, although the number of clear nights was abnormally great, definition was seldom satisfactory.

The instrument employed was the $9\frac{3}{4}$-inch equatorial, bearing negative eyepieces magnifying 218, 300, 400, and 600 diameters, the observers being M. Antoniadi and myself.

1. *The Globe.*

The N. polar cap was not very dark in 1900. No certain traces of the N. temperate band were seen. The great double tropical belt was, however, a very striking feature of the planet, its duplex character being recognised under almost any kind of definition—even when the Cassini division was invisible.* The dark spots occasionally found on this belt were much better seen in 1900 than in 1899. Thus, one of them, having a condensation on each of the components of the belt, was seen in transit over the central meridian on 1900 July $15^{d}\ 11^{h}\ 10^{m}$ G.M.T. Another, much more marked spot, but having no condensation on the N. branch, was central on September $5^{d}\ 7^{h}\ 41^{m}$; and a third, double spot, on September $5^{d}\ 8^{h}\ 33^{m}$. By far the brightest, though yellowest, portion of the globe was the equatorial zone. It was very uneven in tint, and mottled in appearance. A faint dusky band, almost marking the equator, was seen with certainty on July 10 and September 5.

The decreasing luminosity of the globe towards the limb was very marked.

2. *The Rings.*

(*a*) *Outer Ring, A.*—Encke's division could never be seen, though carefully looked for. The indentations along the ansæ, near Cassini's division, were easily seen, and have been noted on many occasions in 1900. As far as could be judged, their structure was triangular; one side of each triangle resting on the inner edge of this Ring.

As usual, Cassini's division seemed grey and not black. Probably there is some matter in it. In fact, Maxwell has shown that the collisions among the particles would tend to widen the several rings. The division seemed still tangent to the globe,

* Dr. Deslandres was recently successful in photographing the double belt with the great 23·6-inch photographic equatorial of Meudon.